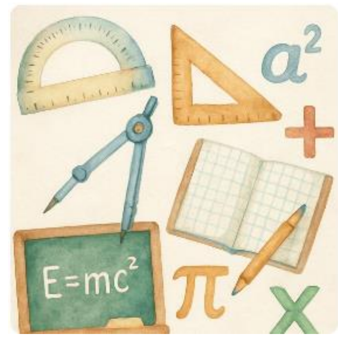
CH X – SYSTEMS OF EQUATIONS, AN INTRODUCTION



Here's an example of a ***system of two equations in two variables***:

$$\begin{aligned} x + y &= 10 \\ 2x - 3y &= 5 \end{aligned}$$

The *two equations* are easy to see, as are the *two variables*. The term *system* refers to the fact that these two equations are tied together — our final solution must be a pair of numbers, one for *x* and one for *y*, satisfying both equations.

❑ *WHAT DOES IT MEAN TO BE A SOLUTION OF A SYSTEM OF EQUATIONS?*

In the system above, the values $x = 8$ and $y = 2$ will satisfy the first equation [8 + 2 = 10 ✓], but will not satisfy the second equation [2(8) – 3(2) = 16 – 6 = 10, not 5]. Therefore, $x = 8$ and $y = 2$ is not a solution of the *system* of equations.

Sticking with the same system of equations above, the values $x = -8$ and $y = -7$ will satisfy the second equation [2(–8) – 3(–7) = –16 + 21 = 5 ✓] but will not satisfy the first equation [–8 + (–7) = –15, not 10]. Therefore, $x = -8$ and $y = -7$ is another pair of numbers which do not constitute a solution of the *system* of equations, either.

But $x = 7$ and $y = 3$ is a solution of the system of equations. Here's why:

7 + 3 = 10 ✓ *and* 2(7) – 3(3) = 14 – 9 = 5 ✓

❑ WHERE CAN SUCH A SYSTEM OF EQUATIONS ARISE?

How does such a system of equations come about? How about a problem like the following:

Find two numbers whose sum is 17 and whose difference is 1.

If you wanted to solve this problem using Algebra, you would proceed something along these lines: First give names to the two unknown numbers; maybe call them g and h.

Then the *sum* part of the problem yields the equation

$$g + h = 17$$

The *difference* part of the problem would imply that

$$g - h = 1$$ [We will assume that $g > h$.]

We now have a **system** of two equations in two variables: $\begin{aligned} g + h &= 17 \\ g - h &= 1 \end{aligned}$

Among the variety of methods available to solve such a system, pay attention to the method required by your instructor.

Homework

1. Consider the system of equations: $\begin{aligned} a + b &= 9 \\ a - b &= 7 \end{aligned}$

   a. Show that $a = 5$ and $b = 4$ <u>is</u> a solution of the first equation, but is <u>not</u> a solution of the system.

   b. Show that $a = 20$ and $b = 13$ <u>is</u> a solution of the second equation, but is <u>not</u> a solution of the system.

   c. Show that $a = 8$ and $b = 1$ <u>is</u> a solution of the system.

2. Try to solve the system $\begin{aligned} u + w &= 12 \\ u - w &= 0 \end{aligned}$ by guessing.

   Now try the system $\begin{aligned} 3x - 17y &= 200 \\ -5x - 12y &= 29 \end{aligned}$ by guessing (just kidding!).

3. Give an example of a system of three equations in three variables.

4. Translate the following word problem to a system of two equations in two variables: *The sum of two numbers is 25 and their product is 150.*

5. Translate the following word problem to a system of two equations in two variables: *The difference of two numbers is 25 and their quotient is 6.*

Solutions

1. a. $5 + 4 = 9$ ✓ But $5 - 4 = 1 \neq 7$.

b. $20 - 13 = 7$ ✓ But $20 + 13 = 33 \neq 9$.

c. $8 + 1 = 9$ ✓ And $8 - 1 = 7$ ✓

2. Both variables = 6, since 6 + 6 = 12, and 6 – 6 = 0.

3. Something like

$$\begin{aligned} 3x - 2y + z &= 9 \\ 5x + y + 10z &= 44 \\ -4x - 5y - z &= -2 \end{aligned}$$

4. $x + y = 25$

$xy = 150$ [Of course, you can use any two variables you'd like.]

5. $a - b = 25$

$\frac{a}{b} = 6$ [We're assuming that $a > b$ and that $b \neq 0$.]

"To educate a man in mind and not in morals is to educate a menace to society."

Theodore Roosevelt
(1858–1919)